

AD-A144 997

NORMAL APPROXIMATION FOR RESPONSE TIME IN A  
PROCESSOR-SHARED COMPUTER SYSTEM MODEL(U) NAVAL  
POSTGRADUATE SCHOOL MONTEREY CA 5 PORNURIYA MAR 84

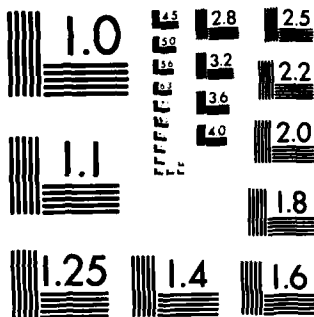
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MICROCOPY RESOLUTION TEST CHART  
NATIONAL BUREAU OF STANDARDS-1963-A

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101 CONTINUE
102 DO 103 J=1,N
    PI(I,(J+1))=GAMMA((FLOAT(N+1)))/GAMMA((FLOAT(N-J+1)))
    PI(I,(J+1))=PI(I,(J+1))*((LAM2/MU2)**J)
103 CONTINUE
    DO 104 I=1,M
        PI((I+1),1)=GAMMA((FLOAT(N+1)))/GAMMA((FLOAT(N-I+1)))
        PI((I+1),1)=PI((I+1),1)*((LAM1/MU1)**I)
104 CONTINUE
        PI(1,1)=0.
        DO 120 I=1,M1
            DO 110 J=1,N1
                PI(1,1)=PI(1,1)+PI(I,J)
110 CONTINUE
120 CONTINUE
        PI(1,1)=1./((PI(1,1)+1.))
        DO 140 I=1,M1
            DO 130 J=1,N1
                IF ((I.EQ.1) .AND. (J.EQ.1)) GO TO 130
                PI(I,J)=PI(1,1)*PI(I,J)
130 CONTINUE
140 CONTINUE
        DO 180 I=1,M1
            DO 170 J=1,N1
                TUI(I,J)=PI(I,J)*LAM1*(M1-I)
170 CONTINUE
180 CONTINUE
        Q1=0.
        DO 200 I=1,M1
            DO 190 J=1,N1
                Q1=Q1+TUI(I,J)
190 CONTINUE
200 CONTINUE
        Q1=1./Q1
        DO 220 I=1,M1
            DO 210 J=1,N1
                TUI(I,J)=TUI(I,J)*Q1
210 CONTINUE
220 CONTINUE
        DO 225 J=1,N1
            NUI(1,J)=0.
225 CONTINUE
        DO 232 I=2,M1
            DO 231 J=1,N1
                NUI(I,J)=TUI((I-1),J)
231 CONTINUE
232 CONTINUE

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235 CONTINUE
      RETURN
      END
C *****
C SUBROUTINE TO SIMULATE RESPONSE TIME AND COMPARISON TO CLT *****
C *****
C *****
      SUBROUTINE SIMRT(M,N,I,LAM1,LAM2,MU1,MU2,X1,A1,B1,C1
      *,PR1,VARP,PHI,VPH,HTXB,HTSD,QH)
      REAL *4 W0,CO,T,LA,M1,LAM2,MU1,XP(6),XJ,XL,XM,BRI(5,6),CRI(5,6)
      *,LJ1(6),LJ2(6),MJ1(6),MJ2(6),XP4,X1(6,6),B1(6,6),C1(6,6)
      *,EXP1,EXP2,EXP3,EXP4,X1(6,6),PR1(7),PH1(7),VARP(7),DMY(7),QH(7)
      *,QU(7),CI(7),PK1(7),PK3(7),NU1(6,6),XBLT,SDLT,HTXB,HTSD
      *,VPH(7),DNY(7),IX1,IX2,M,N,M1,N1,KT,K1(7),K3(7)
      INTEGER IX1,IX2,M,N,M1,N1,KT,K1(7),K3(7)
      IX1 = 253766
      IX2 = 344921
      M1 = M+1
      N1 = N+1
      KT = 300
      DO 10 I=1,M1
        LJ1(I) = (M1-I)*LAM1
        MJ1(I) = MU1*(I-1)
      10 CONTINUE
      DO 11 J=1,N1
        LJ2(J) = (N1-J)*LAM2
        MJ2(J) = MU2*(J-1)
      11 CONTINUE
      C CALL SUBROUTINE TO COMPUTE STEADY-STATE DISTRIBUTION
      CALL SST(M,N,LAM1,LAM2,MU1,MU2,NU1)
      C STD. NORMAL QUANTILES (.10,.25,.50,.75,.90,.95,.99)
      QU(1) = -1.2816
      QU(2) = -0.6745
      QU(3) = 0.6745
      QU(4) = 1.2816
      QU(5) = 1.6449
      QU(6) = 2.3263
      QU(7) = 2.3263
      C CALL SUBROUTINE TO READ CLT MEAN AND VARIANCE
      CALL APPROX(XBLT,SDLT)
      XBLT = XBLT*I
      SDLT = SDLT*I
      C CALL SUBROUTINE TO COMPUTE HEAVY TRAFFIC MEAN AND VARIANCE
      CALL HTA2(T,M,N,LAM1,LAM2,MU1,MU2,HTXB,HTSD)
      HTSD = SQRT(HTSD)
      DO 13 I=1,7
        QI(I) = XBLT+SDLT*QU(I)
      13 CONTINUE
      PRI(1) = 0.
  
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VARP(I) = 0.  
 QH(I) = HTXB+HTSD\*QU(I)  
 PH(I) = 0.  
 VPH(I) = 0.

13 CONTINUE

C CONDITIONAL RESPONSE TIME

DO 25 NI=1,M  
 DO 24 NJ=1,N1  
 RI(NI,NJ) = 0.  
 AR1(NI,NJ) = 0.  
 BR1(NI,NJ) = 0.  
 CR1(NI,NJ) = 0.

24 CONTINUE

DO 25 NI=1,M  
 DO 240 J=1,N1  
 DO 220 K=1,K1  
 W0 = 1.  
 C0 = 0.  
 I0 = 1.  
 J0 = J

C GENERATE EXPONENTIAL ARRIVAL AND DEPARTURE TIME

30 CALL LEXPN(IXI,EXP,6,1,0)

IF ( I0 .NE. M ) GO TO 50  
 EXP1 = 9999.99  
 IF ( I0 .NE. 1 ) GO TO 38

EXP3 = 9999.99  
 GO TO 39

EXP3 = EXP(3)/(MU1\*(I0-1))  
 IF ( J0 .NE. 1 ) GO TO 40  
 EXP2 = EXP(2)/(LJ2(J0)\*(I0+J0-1))  
 XL = EXP(5)/(LJ1(I0+1)+LJ2(J0))\*(I0+J0-1)

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EXP4 = 9999.99  
 XM = EXP3  
 GO TO 110

IF ( J0 .NE. N1 ) GO TO 41  
 IF EXP2 = 9999.99

XL = EXP1  
 EXP4 = EXP(4)/(MU2\*(J0-1))  
 XM = EXP(6)/(MU1\*(I0-1)+MU2\*(J0-1))

40

GO TO 110

EXP2 = EXP(2)/(LJ2(J0)\*(I0+J0-1))  
 XL = EXP(5)/(LJ1(I0+1)+LJ2(J0))\*(I0+J0-1)  
 EXP4 = EXP(4)/(MU2\*(J0-1))  
 XM = EXP(6)/(MU1\*(I0-1)+MU2\*(J0-1))

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50      GO TO 110
      IF ( IO .NE. 1 ) GO TO 55
      EXP1 = EXP(1)/(LJ1(10+1)*(10+J0-1))
      EXP3 = 9999.99
      GO TO 39
55      EXP1 = EXP(1)/(LJ1(10+1)*(10+J0-1))
      EXP3 = EXP(3)/(MU1*(10-1))
      GO TO 35
110     XJ = AMIN1(XL,XM)
      IF ( XJ .LT. W0 ) GO TO 120
      RI(1,J) = RI(1,J)+(CO+W0*(10+J0-1))/(FLOAT(KT))
      XR(K) = CO+W0*(10+J0-1)
      GC TO 220
120     CO = CO+XJ*(10+J0-1)
      W0 = W0-XJ
      IF ( XJ .NE. XL ) GO TO 140
      IF ( EXP1 .GT. EXP2 ) GO TO 130
      IF ( IO .EQ. M ) GO TO 160
      IO = IO+1
      GO TO 160
130     IF ( JO .EQ. N1 ) GO TO 160
      JO = JO+1
      GO TO 160
140     IF ( EXP3 .GT. EXP4 ) GO TO 150
      IF ( IO .EQ. 1 ) GO TO 160
      IO = IO-1
      GO TO 160
150     IF ( JO .EQ. 1 ) GO TO 160
      JO = JO-1
      IX1 = IX1+1
      GO TO 30
220     CONTINUE
230     CONTINUE
      DO 231 LI=1,7
      K1(LI) = 0
      K3(LI) = 0
      CONTINUE
      DO 235 K=1,KT
      AR1(1,J) = AR1(1,J)+(XR(K)-RI(1,J))*2/(FLOAT(KT))
      BR1(1,J) = BR1(1,J)+(XR(K)-RI(1,J))*3/(FLOAT(KT))
      CR1(1,J) = CR1(1,J)+(XR(K)-RI(1,J))*4/(FLOAT(KT))
      IF ( XR(K) .GT. QI(1) ) GO TO 355
      DO 350 LI=1,7
      K1(LI) = K1(LI)+1
      CONTINUE
      GO TO 400
350     CONTINUE
      GO TO 400
355     IF ( XR(K) .GT. QI(2) ) GO TO 360
      DC 357 LI=2,7
  
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357 K1(LI) = K1(LI)+1  
 CCNTINUE  
 GO TO 400  
 360 IF (XR(K) .GT. QI(3) ) GO TO 365  
 DO 363 LI=3,7  
 K1(LI) = K1(LI)+1  
 363 CCNTINUE  
 GO TO 400  
 365 IF (XR(K) .GT. QI(4) ) GO TO 370  
 DO 367 IL=4,7  
 K1(IL) = K1(IL)+1  
 367 CCNTINUE  
 GO TO 400  
 370 IF (XR(K) .GT. QI(5) ) GO TO 375  
 DO 372 KI=5,7  
 K1(KI) = K1(KI)+1  
 372 CCNTINUE  
 GO TO 400  
 375 IF (XR(K) .GT. QI(6) ) GO TO 380  
 DO 377 LK=6,7  
 K1(LK) = K1(LK)+1  
 377 CCNTINUE  
 GO TO 400  
 380 IF (XR(K) .GT. QI(7) ) GO TO 400  
 K1(7) = K1(7)+1  
 400 CCNTINUE  
 235 DO 800 MI=1,KT  
 IF (XR(MI) .GT. QH(1) ) GO TO 710  
 DO 705 MK=1,7  
 K3(MK) = K3(MK)+1  
 705 CCNTINUE  
 GO TO 780  
 710 IF (XR(MI) .GT. QH(2) ) GO TO 720  
 DO 715 MKK=2,7  
 K3(MKK) = K3(MKK)+1  
 715 CCNTINUE  
 GO TO 780  
 720 IF (XR(MI) .GT. QH(3) ) GO TO 730  
 DO 725 MKI=3,7  
 K3(MKI) = K3(MKI)+1  
 725 CCNTINUE  
 GO TO 780  
 730 IF (XR(MI) .GT. QH(4) ) GO TO 740  
 DO 735 MKJ=4,7  
 K3(MKJ) = K3(MKJ)+1  
 735 CCNTINUE  
 GO TO 780

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74C IF (XR(MI).GT. QH(5)) GO TO 750
    DO 745 MKL=5,7
        K3(MKL) = K3(MKL)+1
    CONTINUE
    GC TO 780
745 IF (XR(MI).GT. QH(6)) GO TO 760
    DO 755 MKM=6,7
        K3(MKM) = K3(MKM)+1
    CONTINUE
    GC TO 780
755 IF (XR(MI).GT. QH(7)) GO TO 780
    K3(7) = K3(7)+1
    CONTINUE
    GC TO 780
760 CONTINUE
800 DO 410 KK=1,7
    PK1(KK) = FLOAT(K1(KK))/FLOAT(KT)
    PK3(KK) = FLOAT(K3(KK))/FLOAT(KT)
    CONTINUE
410 DO 420 LM=1,7
    PRI(LM) = PRI(LM)+PK1(LM)*NUL1(I+1,J)
    PHI(LM) = PHI(LM)+PK3(LM)*NUL1(I+1,J)
    CONTINUE
240 CONTINUE
250 DO 425 LL=1,7
    IF (PRI(LL).GT. 1.0) GO TO 200
    GO TO 201
    PRI(LL) = 1.0
    IF (PHI(LL).GT. 1.0) PHI(LL) = 1.0
    DO 423 JJ=1,N1
        DMY(LL) = PRI(LL)*(1-PRI(LL))*NUL1(II+1,JJ)**2/FLOAT(KT)
        VARP(LL) = VARP(LL)+DMY(LL)
        DNY(LL) = PHI(LL)*(1-PHI(LL))*NUL1(II+1,JJ)**2/FLOAT(KT)
        VPH(LL) = VPH(LL)+DNY(LL)
    CONTINUE
423 CONTINUE
424 CONTINUE
425 CONTINUE
490 DO 310 JJJ=1,N1
    XI(1,JJJ) = 0.
    AI(1,JJJ) = 0.
    BI(1,JJJ) = 0.
    CI(1,JJJ) = 0.
    CONTINUE
310 DO 320 JK=1,M
    DO 315 JK=1,N1
        XI((K+1),JK) = RI(IK,JK)

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SO = (SQRT((BS2+AS1)**2-4*(AS1*BS2-AS2*BS1))-BS2-AS1)/2.
SI = -(BS2+AS1+SO)
BI1 = (SO+BS2)/(SO-SI)
BI2 = BS2/(SI-SO)
BI3 = AS2/(SI-SO)
B21 = (SO+AS1)/(SO-SI)
B22 = (SI+BS2)/(SI-SO)
K11 = (SI+AS1)/(SI-SO)
K12 = AS2/(SO-SI)
K21 = BS2/(SO-SI)
K22 = (AS1-BS1)/(SO*SI)
G31 = (AS1-AS2)/(SO*SI)
G32 = (G31*SI+1)/(SO-SI)
B31 = (G31*SI+1)/(SI-SO)
B32 = (G31*SO+1)/(SI-SO)
K31 = (G31*SO+1)/(SI-SO)
K32 = -SM1**2*(BI1**2/(2*SO)+2*BI1*K11/(SO+SI)+K11**2/(2*SI))
Y10 = -SM1**2*(BI1**2/(2*SO)+2*BI1*K12/(SO+SI)+K12**2/(2*SI))
Y11 = -SM1**2*(B21**2/(2*SO)+2*B21*K21/(SO+SI)+K21**2/(2*SI))
Y20 = -SM1**2*(B22**2/(2*SO)+2*B22*K22/(SO+SI)+K22**2/(2*SI))
D10 = BI1*B21/(2*SO)+(K11*B21+BI1*K21)/(SO+SI)+K11*K21/(2*SI)
D20 = BI1*B22/(2*SO)+(K12*B22+BI1*K22)/(SO+SI)+K12*K22/(2*SI)
DS1 = BS1**2*(EXP(2*SO**T)-1)/(2*SO)+K31**2*(EXP(2*SI**T)-1)/(2*SI)
DS2 = 2*G31*B31*(EXP(SO**T)-1)/(SO+2*G31*K31*(EXP(SI**T)-1)/SI)
DS3 = G31**2*(1+2*B31*K31*(EXP((SO+SI)*T)-1)/(SO+SI))
ZT1 = DS2+DS3
DT1 = BS2**2*(EXP(2*SO**T)-1)/(2*SO)+K32**2*(EXP(2*SI**T)-1)/(2*SI)
DT2 = 2*G32*B32*(EXP(SO**T)-1)/(SO+2*G32*K32*(EXP(SI**T)-1)/SI)
DT3 = G32**2*(1+2*B32*K32*(EXP((SO+SI)*T)-1)/(SO+SI))
ZT2 = DT2+DT3
DZ1 = (G31+B31)*EXP(SO**T)+K31*(SI**T)**2**Y10
DZ2 = (G32+B32)*EXP(SO**T)+K32*(SI**T)**2**Y20
DZ3 = 2*SQRT(DZ1/Y10)*SQRT(DZ2/Y20)*Y12
C COMPUTE THE VARIANCE
HTSD = (DZ1+DZ2+DZ3+ZT1*SM1**2+ZT2*SM2**2)*M
C
RETURN
END

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# LIST OF REFERENCES

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